1.) Let $\boldsymbol{A} = \begin{pmatrix} 2 & -4 \\ -1 & 3 \end{pmatrix}$.

(a) Find $\boldsymbol{A}^{-1}$.

(2)

(b) Solve the matrix equation $\boldsymbol{AX} = \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}$.

(4)
(Total 6 marks)

2.) Let $\boldsymbol{A} = \begin{pmatrix} 3 & x \\ -2 & -3 \end{pmatrix}$.

(a) Find the value of x for which $\boldsymbol{A}^{-1}$ does not exist.

(3)

(b) Given that $\boldsymbol{A} = \boldsymbol{A}^{-1}$, find x.

(5)
(Total 8 marks)

3.) The system of linear equations below can be written as the matrix equation $\boldsymbol{MX} = \boldsymbol{N}$.

$$\begin{aligned} x + 6y - 3z &= -1 \\ 4x + 2y - 4z &= 12 \\ x + y + 5z &= 15 \end{aligned}$$

(a) Write down the matrices $\boldsymbol{M}$ and $\boldsymbol{N}$.

(3)

(b) Solve the **matrix** equation $\boldsymbol{MX} = \boldsymbol{N}$.

(3)

(c) Hence write down the solution of the system of linear equations.

(1)
(Total 7 marks)

4.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$.

(3)

(b) Solve $\boldsymbol{A}^{-1}\boldsymbol{X} = \boldsymbol{B}$.

(2)
(Total 5 marks)

5.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & -3 \\ -1 & -1 & 4 \\ 2 & 4 & -3 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$.

(2)

(b) Solve $\boldsymbol{AX} = \boldsymbol{B}$.

(3)
(Total 5 marks)

6.) Let $\boldsymbol{A} = \begin{pmatrix} 9e^{x} & e^{x} \\ e^{x} & e^{3x} \end{pmatrix}$.

(a) Find an expression for det $\boldsymbol{A}$.

(2)

(b) Find the value of x for which $\boldsymbol{A}$ has no inverse. Express your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$.

(5)
(Total 7 marks)

7.) Let $\boldsymbol{A} = \begin{pmatrix} 5 & 1 \\ 6 & 2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix}$.

(a) (i) Find $\boldsymbol{AB}$.

(ii) Write down the inverse of $\boldsymbol{A}$.

(3)

Let $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$.

(b) Solve the matrix equation $\boldsymbol{AX} = \boldsymbol{C}$.

(4)
(Total 7 marks)

8.) A matrix $\boldsymbol{M}$ has **inverse** $\boldsymbol{M}^{-1} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$.

(a) Find $\boldsymbol{M}$.

(3)

(b) Solve the matrix equation $\boldsymbol{MX} = \boldsymbol{B}$, where $\boldsymbol{B} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ and $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$.

(3)
(Total 6 marks)

9.) Let $f(x) = ax^2 + bx + c$ where a, b and c are rational numbers.

(a) The point P(–4, 3) lies on the curve of f. Show that $16a - 4b + c = 3$.

(2)

(b) The points Q(6, 3) and R(–2, –1) also lie on the curve of f. Write down two other linear equations in a, b and c.

(2)

(c) These three equations may be written as a matrix equation in the form $\boldsymbol{AX} = \boldsymbol{B}$,

where $\boldsymbol{X} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

(i) Write down the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$.

(ii) Write down $\boldsymbol{A}^{-1}$.

(iii) **Hence** or otherwise, find $f(x)$.

(8)

(d) Write $f(x)$ in the form $f(x) = a(x - h)^2 + k$, where a, h and k are rational numbers.

(3)
(Total 15 marks)

10.) Let $\mathbf{A} = \begin{pmatrix} 3 & 0 & 1 \\ 2 & -3 & 0 \\ 4 & -2 & 1 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$.

(2)

(b) Let $\boldsymbol{B}$ be a 3 × 3 matrix. Given that $\boldsymbol{AB} + \begin{pmatrix} -3 & 2 & 1 \\ 5 & 3 & 4 \\ -9 & 2 & 10 \end{pmatrix} = \begin{pmatrix} 7 & 6 & -7 \\ 6 & 5 & -8 \\ 1 & 7 & -5 \end{pmatrix}$, find $\boldsymbol{B}$.

(4)
(Total 6 marks)

11.) Let $\boldsymbol{M} = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}$.

(a) Write down the determinant of $\boldsymbol{M}$.

(1)

(b) Write down $\boldsymbol{M}^{-1}$.

(2)

(c) **Hence** solve $\boldsymbol{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$.

(3)
(Total 6 marks)

12.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ 3 & p \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -2 & 1 \\ q & \frac{1}{2} \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$ in terms of p and q.

(2)

(b) Matrix $\boldsymbol{B}$ is the inverse of matrix $\boldsymbol{A}$. Find the value of p and of q.

(5)
(Total 7 marks)

13.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 1 \\ 0 & 2 & -2 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$.

(2)

The matrix $\boldsymbol{B}$ satisfies the equation $\left(\boldsymbol{I} - \frac{1}{2}\boldsymbol{B}\right)^{-1} = \boldsymbol{A}$, where $\boldsymbol{I}$ is the 3×3 identity matrix.

(b) (i) Show that $\boldsymbol{B} = -2(\boldsymbol{A}^{-1} - \boldsymbol{I})$.

(ii) Find $\boldsymbol{B}$.

(iii) Write down det $\boldsymbol{B}$.

(iv) **Hence**, explain why $\boldsymbol{B}^{-1}$ exists.

(6)

Let $\boldsymbol{BX} = \boldsymbol{C}$, where $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

(c) (i) Find $\boldsymbol{X}$.

(ii) Write down a system of equations whose solution is represented by $\boldsymbol{X}$.

(5)

(Total 13 marks)

14.) (a) Given $\boldsymbol{A} = \begin{pmatrix} 7 & 8 \\ 2 & 3 \end{pmatrix}$, find $\boldsymbol{A}^{-1}$.

(2)

(b) **Hence**, solve the system of simultaneous equations.

$$7x + 8y = 1$$
$$2x + 3y = 1$$

(4)
(Total 6 marks)

15.) (a) Write down the inverse of the matrix $\boldsymbol{A} = \begin{pmatrix} 1 & -3 & 1 \\ 2 & 2 & -1 \\ 1 & -5 & 3 \end{pmatrix}$.

(2)

(b) **Hence** solve the simultaneous equations

$$x - 3y + z = 1$$
$$2x + 2y - z = 2$$
$$x - 5y + 3z = 3$$

(4)
(Total 6 marks)

16.) (a) Write down the inverse of the matrix $\boldsymbol{A} = \begin{pmatrix} 1 & -3 & 0 \\ 2 & 0 & 1 \\ 4 & 1 & 3 \end{pmatrix}$.

(b) Hence or otherwise solve

$$x - 3y = 1$$
$$2x + z = 2$$
$$4x + y + 3z = -1$$

(Total 6 marks)

17.) Let $\boldsymbol{A} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$.

(a) Find

(i) $\boldsymbol{A}^{-1}$;

(ii) $\boldsymbol{A}^2$.

(4)

Let $\boldsymbol{B} = \begin{pmatrix} p & 2 \\ 0 & q \end{pmatrix}$.

(b) Given that $2\boldsymbol{A} + \boldsymbol{B} = \begin{pmatrix} 2 & 6 \\ 4 & 3 \end{pmatrix}$, find the value of p and of q.

(3)

(c) Hence find $\boldsymbol{A}^{-1}\boldsymbol{B}$.

(2)

(d) Let $\boldsymbol{X}$ be a 2×2 matrix such that $\boldsymbol{AX} = \boldsymbol{B}$. Find $\boldsymbol{X}$.

(2)
(Total 11 marks)

18.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 18 \\ 23 \\ 13 \end{pmatrix}$ and $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

(a) Write down the inverse matrix $\boldsymbol{A}^{-1}$.

(b) Consider the equation $\boldsymbol{AX} = \boldsymbol{B}$.

(i) Express $\boldsymbol{X}$ in terms of $\boldsymbol{A}^{-1}$ and $\boldsymbol{B}$.

(ii) **Hence**, solve for $\boldsymbol{X}$.

(Total 6 marks)

19.) The matrix $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 0 \\ -3 & 1 & -1 \\ 2 & -2 & 1 \end{pmatrix}$ has inverse $\boldsymbol{A}^{-1} = \begin{pmatrix} -1 & -2 & -2 \\ 3 & 1 & 1 \\ a & 6 & b \end{pmatrix}$.

(a) Write down the value of

(i) a;

(ii) b.

Consider the simultaneous equations

$$x + 2y = 7$$
$$-3x + y - z = 10$$
$$2x - 2y + z = -12$$

(b) Write these equations as a matrix equation.

(c) Solve the matrix equation.

(Total 6 marks)

20.) (a) Write down the inverse of the matrix $\boldsymbol{A} = \begin{pmatrix} 1 & -3 & 1 \\ 2 & 2 & -1 \\ 1 & -5 & 3 \end{pmatrix}$

(b) **Hence** solve the simultaneous equations

$x - 3y + z = 1$

$2x + 2y - z = 2$

$x - 5y + 3z = 3$

(Total 6 marks)

21.) Let $C = \begin{pmatrix} -2 & 4 \\ 1 & 7 \end{pmatrix}$ and $\boldsymbol{D} = \begin{pmatrix} 5 & 2 \\ -1 & a \end{pmatrix}$.

The 2 × 2 matrix $\boldsymbol{Q}$ is such that $3\boldsymbol{Q} = 2\boldsymbol{C} - \boldsymbol{D}$

(a) Find $\boldsymbol{Q}$.

(3)

(b) Find $\boldsymbol{CD}$.

(4)

(c) Find $\boldsymbol{D}^{-1}$.

(2)
(Total 9 marks)

22.) Matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are defined by

$\boldsymbol{A} = \begin{pmatrix} 5 & 1 \\ 7 & 2 \end{pmatrix}$ $\boldsymbol{B} = \begin{pmatrix} 2 & 4 \\ -3 & 15 \end{pmatrix}$ $\boldsymbol{C} = \begin{pmatrix} 9 & -7 \\ 8 & 2 \end{pmatrix}$.

Let $\boldsymbol{X}$ be an unknown 2 × 2 matrix satisfying the equation

$$\boldsymbol{AX} + \boldsymbol{B} = \boldsymbol{C}.$$

This equation may be solved for $\boldsymbol{X}$ by rewriting it in the form

$$\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{D}.$$

where $\boldsymbol{D}$ is a 2×2 matrix.

(a) Write down $\boldsymbol{A}^{-1}$.

(2)

(b) Find $\boldsymbol{D}$.

(3)

(c) Find $\boldsymbol{X}$.

(2)
(Total 7 marks)

23.) Consider the matrix $\boldsymbol{A} = \begin{pmatrix} 5 & -2 \\ 7 & 1 \end{pmatrix}$.

(a) Write down the inverse, $\boldsymbol{A}^{-1}$.

(2)

(b) $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{X}$ are also 2×2 matrices.

(i) Given that $\boldsymbol{XA} + \boldsymbol{B} = \boldsymbol{C}$, express $\boldsymbol{X}$ in terms of $\boldsymbol{A}^{-1}$, $\boldsymbol{B}$ and $\boldsymbol{C}$.

(ii) Given that $\boldsymbol{B} = \begin{pmatrix} 6 & 7 \\ 5 & -2 \end{pmatrix}$, and $\boldsymbol{C} = \begin{pmatrix} -5 & 0 \\ -8 & 7 \end{pmatrix}$, find $\boldsymbol{X}$.

(4)
(Total 6 marks)

24.) Let $M = \begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix}$, where a $\in \mathbb{Z}$.

(a) Find M^2 in terms of a.

(4)

(b) If M^2 is equal to $\begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix}$, find the value of a.

(2)

(c) Using this value of a, find M^{-1} and **hence** solve the system of equations:

$$-x + 2y = -3$$
$$2x - y = 3$$

(6)
(Total 12 marks)

25.) A and B are 2×2 matrices, where $A = \begin{bmatrix} 5 & 2 \\ 2 & 0 \end{bmatrix}$ and $BA = \begin{bmatrix} 11 & 2 \\ 44 & 8 \end{bmatrix}$. Find B.

Working:

Answer:

....

(Total 4 marks)